Anderson Localization of Matter Waves in Tailored Disordered Potentials

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➢ 2D: B. Allard, T. Plisson, T. Bourdel
➢ M. Lewenstein, G.V. Shlyapnikov, D. Delande, B. van Tiggelen

LCFIO / QAG-t team

➢ M. Piraud, S. Lellouch, L. Pezzé, L. Dao, LSP
➢ P. Lugan, B. Hambrecht, T. Koffel
Ultracold Atoms and Quantum Simulators

- **Superfluid-Mott insulator Transition** (Hänsch, Bloch, Esslinger)
- **Tonks-Girardeau gas** (Weiss, Bloch)
- **BEC-BCS crossover** (Jin, Ketterle, Grimm, Salomon)
- **Quantized vortices** in Fermion gases (Ketterle)
- **Spin exchange** (Phillips, Bloch)
- **Anderson localization** (Aspect, Inguscio, Hulet)

Other key phenomena:
- **Quantized vortices**
- **Berezinskii-Kosterlitz-Thouless crossover** (Dalibard, Phillips, Cornell)
Anderson Localization

- interference of quantum paths
- central role of loops

⇒ Absence of diffusion
⇒ exponential localization of wavefunctions

⇒ 1D ≠ 2D ≠ 3D

1D : all states are localized
2D : all states are localized; Lloc grows exponentially with E; marginal dimension for the Anderson transition
3D : mobility edge; finite localization for $l_B < \sim \lambda/2\pi$
Disordered Quantum Gases

Dynamical localization
- Zoller et al., PRA (1992)
- Raizen et al., PRL (1994)
- Garreau et al., PRL (2008 and 2010)

Anderson localization
- 1D th: Lewenstein/Zoller et al., PRL (2003); LSP et al., PRL (2007)
- 2D/3D th: Kuhn et al., PRL (2005), NJP (2007); van Tiggelen et al., PRL (2008); LSP et al. (2011)
- Towards 2D AL exp: Aspect/Bouyer/Bourdel et al., PRL (2010)
- Towards 3D AL exp: DeMarco et al. (2011), Aspect/Bouyer/Josse et al. (2011)

Interplay of interactions and disorder
- Large body of theoretical work; Giamarchi & Schulz (1988), Fisher et al. (1989)

Disordered quantum gases under control
Laurent Sanchez-Palencia1* and Maciej Lewenstein2*

When attempting to understand the role of disorder in condensed-matter physics, we face considerable experimental and theoretical difficulties, and many questions are still open. Two of the most challenging ones—debated for decades—concern the effect of disorder on superconductivity and quantum magnetism. We review recent progress in the field of ultracold atomic gases, which should pave the way towards the realization of versatile quantum simulators, which help solve these questions. In addition, ultracold gases offer original practical and conceptual approaches, which open new perspectives to the field of disordered systems.

Nature Physics | Vol 6 | February 2010 | www.nature.com/naturephysics
1. Anderson localization of 1D matter waves
   ➢ J. Billy et al., Nature 453, 891 (2008)

2. Effect of correlations in 1D speckle potentials

3. Effect of correlations in 1D speckle potentials
   ➢ M. Piraud et al., arXiv:1104.2314
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Controlled Disorder in Quantum Gases

Speckles: an original class of disorder

- $E(r)$ is a Gaussian random process
- $V(r) \propto |E(r)|^2 - \langle |E|^2 \rangle$ is not Gaussian
- $C_n(r_1, \ldots, r_n) = \langle V(r_1) \ldots V(r_n) \rangle$ all
determined by $C_E(r) = \langle E(r)^*E(0) \rangle$

Experimental control

$$C_2(z) = \langle V(z)V(0)\rangle - \langle V \rangle^2$$

- $V_R \propto I_L / \Delta$
- $\sigma_R$: correlation length
  (depends only on the intensity profile of the ground-glass plate)

D. Clément et al.,
Anderson Localization of a Matter Wave

J. Billy et al., Nature 453, 891 (2008)

\[ n(z, t \to \infty) \approx \int dE \, D_E(E) \, P(z, t \to \infty | E) \]

see also G. Roati et al., Nature 453, 895 (2008)

superposition of matter waves with a broad energy distribution
Anderson Localization of a Matter Wave

J. Billy et al., Nature 453, 891 (2008)
Anderson Localization of a Matter Wave

J. Billy et al., Nature 453, 891 (2008)

fair agreement between theory and experiments
1. Anderson localization of 1D matter waves
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\[ \gamma(E) \simeq \left( \frac{m^2}{2\hbar^2 p_E^2} \right) \tilde{C}(2p_E) \]

FT of the correlation function of the disorder

Localization in a Speckle Potential

\[ \gamma(E) \sim \left( \frac{m^2}{2\hbar^2 p_E^2} \right) \tilde{C}(2p_E) \]

FT of the correlation function of the disorder

Correlation function of a 1D speckle potential

\[ \tilde{C}(p) = \pi V_R^2 \sigma_R \left[ 1 - \left| p \right| \sigma_R / 2\hbar \right] \]

\[ \gamma(E) \simeq \left( \frac{m^2}{2\hbar^2} \frac{p^2}{E} \right) \tilde{C}(2p_E) \]

Correlation function of a 1D speckle potential

\[ \tilde{C}(p) = \pi V_R^2 \sigma_R \left[ 1 - \frac{|p| \sigma_R}{2\hbar} \right] \]

\[ C(2p_E) \]

\[ \gamma \neq 0 \quad \gamma \simeq 0 \]

FT of the correlation function of the disorder

effective mobility edge

Localization in a Speckle Potential

\[ n(z, t \to \infty) \sim \int dE \, D_E(E) \, P(z, t \to \infty | E) \]

exponential decay

with \( L_{\text{loc}}(E) = 1/\gamma(E) \)

\[ n(z) \sim 1/|z|^{\beta} \]

\[ \beta \approx 2.01 \pm 0.03 \]

algebraic localization

J. Billy et al., Nature 453, 891 (2008)
Localization in a Speckle Potential

\[ \gamma(E) \approx \left( \frac{m^2}{2\hbar^2 p_E^2} \right) \tilde{C}(2p_E) \]

No contradiction with the theorem that all states are localized in 1D!

calculation beyond the Born approximation

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Localization in Tailored Disorder

M. Piraud et al., arXiv:1104.2314
see also M. Plodzien and K. Sacha, arXiv:1103.3424

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Localization in Tailored Disorder

\[ \gamma(E) \simeq \left( \frac{m^2}{2\hbar^2 P_E^2} \right) \tilde{C}(2p_E) \]

- localization decreases with the particle energy
- an intuitive result ...
Localization in Tailored Disorder

Localization can increase with the particle energy

\[ \gamma(E) \simeq \left( \frac{m^2}{2\hbar^2 p^2_E} \right) \tilde{C}(2p_E) \]

⇒ localization can increase with the particle energy
⇒ a counter-intuitive effect!
Localization in Tailored Disorder

How to observe the increase of localization with energy?

Does the effect show up also for $d > 1$?

Why is it an interesting effect?

⇒ localization can increase with the particle energy
⇒ a counter-intuitive effect!
How to observe the increase of localization with energy?
How to observe the increase of localization with energy?

![Diagram](image)

Graph showing the relationship between $\gamma(E)/\sigma_R$ and $k_E\sigma_R$. The graph has a peak at $k_E\sigma_R = 3$.
Localization in Tailored Disorder

M. Piraud et al., arXiv:1104.2314

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fit $P_\infty(z|E)$ to data and extract $\gamma(E)$
Localization in Tailored Disorder

M. Piraud et al., arXiv:1104.2314

How to observe the increase of localization with energy?

- Fit $P_\infty(z|E)$ to data and extract $\gamma(E)$
Does the effect show up also in $d > 1$?
Does the effect show up also in $d > 1$ ?

$$\gamma(E) \simeq \left( \frac{m^2}{2\hbar^2}\tilde{p}_E^2 \right) \tilde{C}(2\tilde{p}_E)$$
Does the effect show up also in $d > 1$?

Self-consistent theory of AL (Vollhardt & Wölfe, 1980)

$$\gamma(E) = F_d(k_E, l_B)$$

$F_d$ decreases with $k_E$ and $l_B$

**calculation of $l_B$**

- transport theory (weak disorder)
- complicated function of $k_E$ and $C(k)$
- In general, $l_B$ increases with $k_E$
Localization in Tailored Disorder

Does the effect show up also in d> 1?

Self-consistent theory of AL (Vollhardt & Wölfe, 1980)

\[ \gamma(E) = F_d(k_E, l_B) \]

- \( F_d \) decreases with \( k_E \) and \( l_B \)

calculation of \( l_B \)

- transport theory (weak disorder)
- complicated function of \( k_E \) and \( C(k) \)
- In tailored disorder, \( l_B \) can decrease with \( k_E \)

\[ \gamma(E) \simeq \left( \frac{m^2}{2\hbar^2p_E^2} \right) \tilde{C}(2p_E) \]
Does the effect show up also in $d > 1$?

$\gamma(E)$ can increase with $E$ for a sufficiently large mask!
Why is it an interesting effect?

energy $E$ (in the disorder)
**Localization in Tailored Disorder**

Why is it an interesting effect?

- energy $E$ (in the disorder)

Numerical simulation

- absence of diffusion
- exponential localization

2D Anderson localization?
Why is it an interesting effect?

- energy $E$ (in the disorder)

**Numerical simulation**
- absence of diffusion
- exponential localization

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**NOT ANDERSON LOCALIZATION**
- purely classical simulation
- here, classical localization (sub-percolating energy)

⇒ The classical localization length always increases with $E$, even in tailored disorder!
Conclusions and Perspectives

- Anderson localization of 1D matter waves
  - exponential localization
  - direct access to $\gamma(k_E) = L_{loc}(k_E)^{-1}$

- Effect of finite-range correlations in speckle potentials
  - strong effect on $\gamma(k_E)$ at $k_E = \sigma_R^{-1}$
  - algebraic localization

- Anderson localization in tailored disorder
  - easy realization with speckle potentials
  - counter-intuitive behavior of $\gamma(k_E)$
  - 1D, 2D and 3D (?)
  - discrimination of classical versus quantum localization

Perspectives

- Self-consistent theory of AL in anisotropic 2D and 3D disorder
- AL in tailored 3D disorder

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